

EC 320: Introduction to Econometrics

Lecture Notes

David M. Hall
University of Oregon

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1 Session Overview

- Date:
- Topic:
 - Chapters (Dougherty):
- Goals/Objectives:
 1. One
 2. Two
 3. Three

1.1 Before Class:

2 During Class:

2.1 Welcome

Today we will cover the following:

- Perform a t test on regression outputs.
- Create CIs based on a regression output.
- Understand how to perform an F-Test

2.1.1 Warm Up:

Example 2.1. Suppose we are testing whether or not the mean μ of a distribution with a **known** standard deviation is equal to 58. That is:

$$H_0 : \mu = 58$$

Given a standard deviation of 1.75, what would the acceptance region be at the:

- 5 percent level
- 10 percent level

Answer: $58 \pm 1.96 * 1.75$ and $58 \pm 1.645 * 1.75$

2.2 Content Section #1:

Time:

Some things to recall from what we've discussed:

- The estimate $\hat{\beta}_1$ comes from a **distribution**, assumed normal.
- The normal distribution is described by a mean and a variance, with population parameters μ and σ^2 .
 - We do not know those population parameters, but we can certainly estimate them.

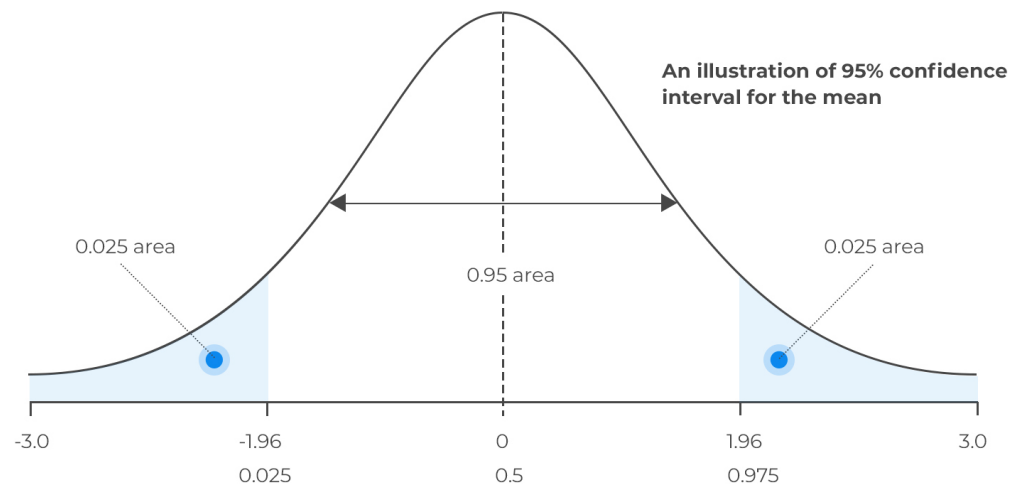
In our review sessions, we covered the z statistic where we can test hypotheses regarding the mean of a normal distribution based on the *known* variance. We briefly discussed what happens when we do *not* know the variance. That is the case we will be discussing today: since we do not know the variance of our estimate's distribution, we must estimate it; and, since we estimate it, we cannot do a standard z test and must, instead, perform a t test.

I will assume you recall the basics about hypothesis testing, but for a brief recall just note that:

- We are testing some hypothesis, assuming the null H_0 unless we have enough proof to reject the null in favor of the alternative H_a .
- There exist one-sided and two-sided tests.
 - One-sided tests involve explaining whether our estimate is above/below a certain testable threshold (e.g. the mean of the estimate above/below some number).
 - Two-sided tests are concerned with pinning down whether the estimate is equal to a certain value (e.g. the mean of the estimate is equal to some number).
- Tests have confidence levels that relate to how much of the distribution is within that level. For example, a 5% significance level indicates that 95% of the density is within the interval that you are describing.



95% Interval



- We decide whether we reject or fail to reject the null hypothesis in one of three ways:
 1. We can use the confidence interval, rejecting if the estimate is outside of the interval.

$$CI : \bar{X} \pm z_{\alpha} * s.d.(X)$$

2. We can decide based on the z – *stat* which is obtained via:

$$z = \frac{\bar{X} - \mu_0}{s.d.}$$

3. We can use the p -value (rejecting if $p < p_{cv}$ where:

$$p_{cv} = 1 - cdf(z_{1-\alpha})$$

With that being said, let's describe in each of these ways how t tests will differ. What we are testing in particular is usually a test of whether our regression coefficient is equal to either one we are testing in theory, or equal to zero. That is:

$$H_0 : \beta_1 = \beta_1^0$$

where β_1^0 is some hypothesized value for the coefficient.

We can reject or fail to reject that hypothesis based on our same three criterion: is the t – *stat* we get above the t critical value? Does the estimated coefficient lie within the bounds of the confidence interval using the hypothesized value? Is the p – *value* lower than the critical value of p ?

Let's start by obtaining and comparing our t statistic. Recall that the only thing that has changed going from z testing to t is that the standard deviation (the population one) is unknown and we are using an estimate. Therefore, do not be alarmed that all we do to go from z to t is replace $s.d.$ with our estimate of it, $s.e.(\bar{X})$.

$$z = \frac{\bar{X} - \mu_0}{s.d.} \rightarrow t = \frac{\bar{X} - \mu_0}{s.e.(\bar{X})}$$

We will reject if our t value is below the critical value. How do we get the critical value? Great question. Let's take a look at the t and z charts to show how they are different.

Students should notice:

- Degrees of freedom: what is this? ($df = n - k$ where we have k parameters)
- Note the pattern: as $df \rightarrow \infty$ then t gets lower — easier threshold to pass!
- z is the limiting distribution of t

The second way we can reject/fail to reject is through seeing if the estimated coefficient is within our confidence interval. Recall where the CIs come from:

- For the two sided test, we reject the null if $t > t_{cv}$ or $t < -t_{cv}$. So another way we can write this would be:

$$\text{Fail to reject } H_0 \text{ if } -t_{cv} < t < t_{cv}$$

but since $t = \frac{\bar{X} - \mu_0}{s.e.(\bar{X})}$ then we can create a confidence interval that is associated only with our $\bar{X}$ by multiplying everything by $s.e.(\bar{X})$ and adding μ_0 :

$$\text{Fail to reject } H_0 \text{ if } \mu_0 - t_{cv} \cdot s.e.(\bar{X}) < \bar{X} < \mu_0 + t_{cv} \cdot s.e.(\bar{X})$$

The third way is through obtaining the p – value of our test statistic. If we know our t – statistic we can see what the "probability" of getting that particular statistic is based on the degrees of freedom.

First, obtain your degrees of freedom; this is the number of observations n minus one.

Second, you'll need to know your t – stat associated with your estimate. Find your corresponding t – stat on the table — it may be between a few different values, just pick one that suits it best.

Find the α associated with that t . It'll either be at the top where it denotes "cum. prob" with $t_{1-\alpha/2}$ or you can go all the way to the bottom to get your probability from 1 minus your confidence level.

2.3 Activity #1:

Write out a chart for yourself, six boxes. Two rows, three columns. Name the rows z and t , name the columns *test-stat*, *confidence interval* and *p-value*. Write out for yourself the formula for each.

If Time Allows

Suppose you run the following regression with $n = 101$ observations.

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i \quad (1)$$

The output is as follows:

	β_0	β_1
Estimate	1	3
S.E.	0.5	1.52

What is the t – *statistic* associated with a null hypothesis of $\beta_1 = 0$? What about for $\beta_0 = 0$?

$$t = \frac{\bar{X}}{s.e.(\bar{X})}$$

- What is the critical value that our t stat needs to be greater than assuming 95% two-sided confidence?? ^a
- What is your rejection criterion?
- What is the confidence interval?
- What is the rejection decision?
- What is the decision if $n = 1002$?

$$^a t(100, t_{0.975}) = 1.984$$

2.4 Content Section #2:

Time:

So far, we have used the t -test to assess whether an individual coefficient (typically β_1) is statistically different from zero. But what about the regression model as a whole? Does it explain a meaningful portion of the variation in the dependent variable?

This is the purpose of the **F-test for overall significance** or **goodness of fit**. The null hypothesis for the F-test is:

$$H_0 : \beta_0 = \beta_1 = 0$$

In our simple regression model:

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

this means we are testing whether the model has any explanatory power whatsoever — that is, whether all variation in Y is just random noise (i.e., not explained by X or the intercept).

The F-statistic is computed as:

$$F = \frac{\text{Explained Mean Square (ESS per regressor)}}{\text{Residual Mean Square (RSS per residual df)}} = \frac{ESS/(k-1)}{RSS/(n-k)}$$

In simple regression, $k = 2$ (intercept and slope), so:

$$F = \frac{ESS}{1} \cdot \frac{1}{RSS/(n-2)} = \frac{ESS}{RSS/(n-2)}$$

We can relate this to the R^2 as follows:

$$F = \left(\frac{R^2}{1 - R^2} \right) (n - 2)$$

This F-statistic follows an F distribution with degrees of freedom $(1, n - 2)$.

Interpretation

- A large F value suggests the model explains a significant amount of variation in Y .
- A small F value implies that the model does not improve our understanding of Y beyond simply using the mean.

If Time Allows

Proof: In Simple Regression, $F = t^2$

We will show that in simple regression, the F-statistic for overall significance is equivalent to the square of the t -statistic used to test $H_0 : \beta_1 = 0$.

Recall the following definitions:

- Total Sum of Squares (TSS):

$$TSS = \sum (Y_i - \bar{Y})^2$$

- Explained Sum of Squares (ESS):

$$ESS = \sum (\hat{Y}_i - \bar{Y})^2$$

- Residual Sum of Squares (RSS):

$$RSS = \sum (Y_i - \hat{Y}_i)^2 = \sum \hat{u}_i^2$$

Let us now simplify ESS using the regression prediction formula. Recall:

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i \quad \text{and} \quad \bar{Y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{X}$$

Then:

$$\hat{Y}_i - \bar{Y} = (\hat{\beta}_0 + \hat{\beta}_1 X_i) - (\hat{\beta}_0 + \hat{\beta}_1 \bar{X}) = \hat{\beta}_1 (X_i - \bar{X})$$

Substitute into ESS:

$$ESS = \sum (\hat{Y}_i - \bar{Y})^2 = \sum [\hat{\beta}_1 (X_i - \bar{X})]^2 = \hat{\beta}_1^2 \sum (X_i - \bar{X})^2$$

Now recall that the F-statistic in simple regression is:

$$F = \frac{ESS}{RSS/(n-2)} = \frac{\hat{\beta}_1^2 \sum (X_i - \bar{X})^2}{\sum \hat{u}_i^2 / (n-2)}$$

We also know that the variance of $\hat{\beta}_1$ is estimated as:

$$SE(\hat{\beta}_1)^2 = \frac{\sum \hat{u}_i^2 / (n-2)}{\sum (X_i - \bar{X})^2}$$

Take the reciprocal and multiply:

$$F = \frac{\hat{\beta}_1^2}{SE(\hat{\beta}_1)^2} = t^2$$

Therefore, in **simple regression**, the F-test for overall model significance is exactly equivalent to squaring the t -statistic for testing whether $\beta_1 = 0$.

Summary Table

Test Type	Formula (Simple Regression)
t -test	$t = \frac{\hat{\beta}_1}{SE(\hat{\beta}_1)}$
F -test	$F = \left(\frac{R^2}{1-R^2} \right) (n-2)$
Relationship	$F = t^2$

How to Use the F Table and Select the Correct Confidence Level

Once you’ve calculated the F -statistic, you’ll want to compare it to the critical value from the F -distribution table to decide whether to reject the null hypothesis.

Here’s how to choose the correct critical value:

- Look for the row corresponding to the **numerator degrees of freedom**, which is $df_1 = k - 1$.
- Look for the column corresponding to the **denominator degrees of freedom**, which is $df_2 = n - k$.
- Use the significance level α that matches your hypothesis test — usually $\alpha = 0.05$ for a 95% confidence level.

In simple regression:

$$df_1 = 1, \quad df_2 = n - 2$$

So, if you have 101 observations, you would look up $F_{1,99}$ at the 5% level. The value will be close to 3.94 (you can confirm using a table or software).

We reject if $F > F_{cv}$.

2.5 Activity #2 (Optional):

Time:

2.6 Summary Activity (5 Q’s of Reflection):

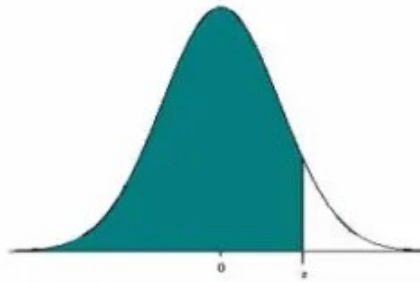
1.

3 Be Able To

- Write out a be able to - separate but related to the goals of this lecture.

4 Key Words

- **Fill in Here**

Table of Standard Normal Probabilities for Positive Z-scores

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
Z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

	DF1	$\alpha = 0.05$																		
DF2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	Inf	
1	161.45	199.5	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	249.05	250.1	251.14	252.2	253.25	254.31	
2	18.513	19	19.164	19.247	19.296	19.33	19.353	19.371	19.385	19.396	19.413	19.429	19.446	19.454	19.462	19.471	19.479	19.487	19.496	
3	10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123	8.7855	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.572	8.5494	8.5264	
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.041	5.9988	5.9644	5.9117	5.8578	5.8025	5.7744	5.7459	5.717	5.6877	5.6581	5.6281	
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725	4.7351	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.4314	4.3985	4.365	
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.099	4.06	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7398	3.7047	3.6689	
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.866	3.787	3.7257	3.6767	3.6365	3.5747	3.5107	3.4445	3.4105	3.3758	3.3404	3.3043	3.2674	3.2298	
8	5.3177	4.459	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472	3.2839	3.2184	3.1503	3.1152	3.0794	3.0428	3.0053	2.9669	2.9276	
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7872	2.7475	2.7067	
10	4.9646	4.1028	3.7083	3.478	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782	2.913	2.845	2.774	2.7372	2.6996	2.6609	2.6211	2.5801	2.5379	
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.948	2.8962	2.8536	2.7876	2.7186	2.6464	2.609	2.5705	2.5309	2.4901	2.448	2.4045	
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.3842	2.341	2.2962	
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	2.671	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2966	2.2524	2.2064	
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6022	2.5342	2.463	2.3879	2.3487	2.3082	2.2664	2.2229	2.1778	2.1307	
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876	2.5437	2.4753	2.4034	2.3275	2.2878	2.2468	2.2043	2.1601	2.1141	2.0658	
16	4.494	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.1058	2.0589	2.0096	
17	4.4513	3.5915	3.1968	2.9647	2.81	2.6987	2.6143	2.548	2.4943	2.4499	2.3807	2.3077	2.2304	2.1898	2.1477	2.104	2.0584	2.0107	1.9604	
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	2.0166	1.9681	1.9168	
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	2.3779	2.308	2.2341	2.1555	2.1141	2.0712	2.0264	1.9795	1.9302	1.878	
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.599	2.514	2.4471	2.3928	2.3479	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.9464	1.8963	1.8432	
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.366	2.321	2.2504	2.1757	2.096	2.054	2.0102	1.9645	1.9165	1.8657	1.8117	
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419	2.2967	2.2258	2.1508	2.0707	2.0283	1.9842	1.938	1.8894	1.838	1.7831	
23	4.2793	3.4221	3.028	2.7955	2.64	2.5277	2.4422	2.3748	2.3201	2.2747	2.2036	2.1282	2.0476	2.005	1.9605	1.9139	1.8648	1.8128	1.757	
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002	2.2547	2.1834	2.1077	2.0267	1.9838	1.939	1.892	1.8424	1.7896	1.733	
25	4.2417	3.3852	2.9912	2.7587	2.603	2.4904	2.4047	2.3371	2.2821	2.2365	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.8217	1.7684	1.711	
26	4.2252	3.369	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655	2.2197	2.1479	2.0716	1.9898	1.9464	1.901	1.8533	1.8027	1.7488	1.6906	
27	4.21	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501	2.2043	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7851	1.7306	1.6717	
28	4.196	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.236	2.19	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7689	1.7138	1.6541	
29	4.183	3.3277	2.934	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229	2.1768	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.7537	1.6981	1.6376	
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107	2.1646	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.7396	1.6835	1.6223	
40	4.0847	3.2317	2.8387	2.606	2.4495	2.3359	2.249	2.1802	2.124	2.0772	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.6373	1.5766	1.5089	
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.097	2.0401	1.9926	1.9174	1.8364	1.748	1.7001	1.6491	1.5943	1.5343	1.4673	1.3893	
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.175	2.0868	2.0164	1.9588	1.9105	1.8337	1.7505	1.6587	1.6084	1.5543	1.4952	1.429	1.3519	1.2539	
Inf	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799	1.8307	1.7522	1.6664	1.5705	1.5173	1.4591	1.394	1.318	1.2214	1	